This application note reviews noise parameter measurements used to characterize transistors and amplifiers at Modelithics Inc. Noise figure and noise parameter theory is reviewed briefly following with a description of the basic test instrumentation setup and calibration procedures used for noise parameter measurements along with an example.

DEFINITIONS AND THEORY

The formulations in this note were derived from multiple sources. The noise figure $F$ of a device or component is described by the following relations:

$$F = \frac{S_{in}/N_{in}}{S_{out}/N_{out}}$$  
(always > 1)

$$= 1 + \frac{N_a}{G N_{in}}$$ with $N_{in} = k T_0 B \ (T_0 = 290^\circ K)$  

Noise Figure (dB) = NF (dB) = 10 log(F)  

where $S_{in}$ ($N_{in}$) and $S_{out}$ ($N_{out}$) are the signal (noise) levels at the input and output of the device, respectively, and $N_a$ is the noise added by the device itself, $G$ is the gain of the device, $B$ is the system bandwidth and $k$ is Boltzman’s constant ($1.38 \times 10^{-23}$). Noise figure is of particular interest to receiver designers as the degradation in the minimum detectable signal can be estimated as:

$$MDS(dBm) = -174 + 10 log(B) + NF(dB) + \text{Required SNR}_{dB}$$  

This approximation assumes the background or ambient (= $k T B$) noise is that due to a passive device held at $T = 290^\circ K$, and the required $\text{SNR}_{dB}$ represents the minimum signal to noise ratio for acceptable system performance.

The noise parameters describe how the noise figure varies with the source impedance $Z_s$, the source admittance $Y_s$, or the source reflection coefficient $\Gamma_s$. Consider Figure 1. There are various formulations for noise fig-

LAWRENCE P. DUNLEAVY
Modelithics Inc., Tampa, FL
ure in terms of noise parameters. Some of the popular forms are summarized in Equations 4 through 7.

\[ Y_{\text{sopt}} = G_{\text{on}} + jB_{\text{on}} \]  

4 Noise Parameters:  
\[ G_{\text{on}}: \text{Equivalent Noise Conductance} \]  
\[ R_{\text{on}}: \text{Optimum Noise Resistance} \]  
\[ X_{\text{on}}: \text{Optimum Noise Reactance} \]  
\[ Z_{\text{sopt}} = R_{\text{on}} + jX_{\text{on}} \]  

\[ F = F_{\text{min}} + \frac{4R_{\text{on}}}{Z_0} \left[ \left( \frac{1}{1 - \left| \Gamma_{\text{sopt}} \right|^2} \right) \right] \]  

4 Noise Parameters:  
\[ R_{\text{on}}: \text{Equivalent Noise Reduction} \]  
\[ F_{\text{min}}: \text{Minimum Noise Figure} \]  
\[ G_{\text{on}}: \text{Optimum Noise Susceptance} \]  
\[ B_{\text{on}}: \text{Optimum Noise Susceptance} \]  

All of the above forms provide a description of noise figure in terms of four (4) noise parameters and the source impedance, admittance or reflection coefficient, depending on the form used. One common parameter is the minimum noise figure \( F_{\text{min}} \), which will be achieved at some specific optimum (complex) impedance \( (Z_{\text{opt}}) \), admittance \( (Y_{\text{opt}}) \) or reflection coefficient \( (\Gamma_{\text{opt}}) \). Therefore, in addition to \( F_{\text{min}} \) alone or two of the other parameters being the equivalent noise resistance \( R_{\text{on}} \), noise conductance \( G_{\text{on}} \) or the terminal invariant parameter \( N \) depending on the formulation. The term terminal invariant implies that \( N \) is invariant to a transformation through a lossless passive network, that is a reference plane change. It should also be noted that there are other noise parameter formulations in addition to those listed in Equations 4 through 7.

One of the common applications of noise parameters is for low noise amplifier (LNA) design. Typically an LNA is used at the front-end of a receiver to improve the noise figure of the receiver or essentially boost the signal, while adding a low amount of noise to the signal. In addition to its noise figure, the gain of the LNA (and correspondingly the transistors used to make up the LNA) is also important. To better understand this, the following equation can be used to calculate the total noise figure of a cascade connection of three different two-port devices with gains \( G_i \) and noise figure \( F_i \) (\( i = 1, 2, 3 \)).

\[ F_{\text{TOT}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} \]  

If an LNA, with high gain \( G_1 \) and low noise figure \( F_1 \), is the first device, then the system noise figure \( F_{\text{TOT}} \) can remain low even if the second and third devices have a much higher noise figure.

The gain used in most noise calculations, including the cascade noise figure (Equation 8), is the available gain, which can be expressed in terms of S-parameters as follows:

\[ G_A = \frac{\left| S_{21} \right|^2 (1 - \left| \Gamma_{\text{sopt}} \right|^2)}{\left| 1 - S_{11} \right|^2 (1 - \left| S_{22} \right|^2)} \]  

The available gain assumes that the load port is terminated in a conjugate match for a given source reflection coefficient \( \Gamma_{\text{sopt}} \). The associated gain is often tabulated along with noise parameters and is simply the available gain from Equation 9 for the particular case of \( \Gamma_{\text{sopt}} = \Gamma_{\text{sopt}} \).

Another set of parameters often plotted are the maximum stable gain, MSG, and maximum available gain, MAG. Often, amplifiers or transistors are unconditionally stable over a certain frequency range and conditionally stable at other frequencies. For frequencies where the device is potentially unstable (with stability factor \( K < 1 \)), the maximum stable gain is defined as the highest realizable gain with passive terminations, after the device is stabilized with cascaded resistance to border line stability; that is to bring about the condition \( K = 1 \). MSG is given by:

\[ MSG = \frac{\left| S_{21} \right|}{\left| S_{12} \right|} \]  

The maximum available gain at frequencies where \( K > 1 \) (unconditionally stable) is given by:

\[ MAG = \frac{\left| S_{21} \right|}{\left| S_{12} \right|} (K - \sqrt{K^2 - 1}) \]  

Hence, MSG and MAG numbers in decibels give the amplifier designer a measure of the maximum gain realizable through impedance matching of the amplifier or transistor. Of course the conditions for matching the input for maximum gain and minimum noise figure may be conflicting and a trade-off between these two may be required. While outside the scope of this note, the plotting of noise figure circles and available gain circles can often be used to aid the designer in choosing the best compro-
mise in matching impedance taking noise and gain into consideration.2

TEST CONFIGURATION AND CALIBRATION

A basic test configuration, used to perform combined noise parameter and S-parameter testing, is shown in Figure 2. The network analyzer is needed to perform S-parameters of the DUT, which are required for design analysis along with the noise parameters. The network analyzer is also needed to make measurements that are required for calibration of the noise parameter test system.

This test system is referred to as an “NP5” system from Maury Microwave4 and uses a hardware setup and measurement method originally developed by Adamian5 and commercialized by ATN Microwave. The system consists of a noise figure measurement system (such as HP 8971C/HP8970B combination or alternative), a network analyzer (such as HP8510C or alternative), a mismatched noise source (MNS) and a remote receiver module (RRM). The MNS and RRM each contain a switch that is used to select either the S-parameter measurement mode or the noise parameter measurement mode. They each also contain a bias tee for applying bias to the input and output of the device under test (DUT). The MNS is generally a solid-state tuner capable of presenting multiple different values of \( \Gamma_s \) to the DUT, along with the ability to have a “50 \( \Omega \)” thru state that allows the noise source to be connected to the DUT through essentially a transmission line. With the RRM switch in the noise measurement position, the RRM includes a low noise amplifier in the path to the noise receiver to improve the measurement receiver noise figure.

In the S-parameter measurement mode the system is calibrated using thru-reflect-line (TRL) or alternative high accuracy calibration approaches.6,7 To calibrate and operate the noise parameter measurement system, the Maury ATS software4 is used to perform a series of steps that may be summarized as follows:

1. Perform a two-port S-parameter calibration to establish measurement reference planes at the input and output of the DUT. Store these calibration coefficients in a selected calibration kit file.

2. Perform a one-port short-open-load S-parameter calibration at the position of the noise source, with a thru device connected in place of the DUT. Store these calibration coefficients in a second selected calibration kit file.

3. Calculate the S-parameters of the MNS thru path from the noise source to the DUT. The Maury ATS software does this automatically using the calibration information from steps 1 and 2; it also measures the noise source reflection coefficient in the hot/biased on state and the cold/biased off state.

4. Perform a tuner characterization. The software uses the calibration information from step 1 to measure and store hundreds of different \( \Gamma_s \) values that can be presented at each frequency to the DUT by the MNS during subsequent measurements.

5. Perform a noise calibration. With a thru connected in place of the DUT and the system switches set to noise measurement position, the ATS software controls the instruments to record the received noise power for the MNS thru state with the noise source diode biased on and off, and for several different \( \Gamma_s \) values achieved with the MNS for the case of the diode biased off.

6. The ATS software utilizes the information from the previous steps along with the algorithm developed by Adamian7 to calculate and store the noise parameters of the receiver along with other system information.

7. Once calibration is complete, the DUT is connected and the S-parameters and noise parameters of the DUT are measured in sequence (usually S-parameters, then noise parameters). Post processing and noise parameter data smoothing is sometimes needed and is provided for in the Maury ATS software.

EXAMPLE RESULTS

In the following, the data taken on samples from the Mini-Circuits SAV amplifier series will be used. Figure 3 shows the picture of a device sample as mounted in a coplanar waveguide test fixture setup for RF wafer probe testing. TRL standards, fabricated with the same ground-signal-ground test interface, were used along with the NIST Multical method8 to establish the measurement reference planes at the locations indicated. These same refer-

![Fig. 2 Maury Microwave “NP5” noise and S-parameter test systems.](image1)

![Fig. 3 Photograph of a mounted SAV series device showing reference planes.](image2)
ence planes were established for noise parameter measurements using Maury ATS software. Figure 4 shows examples of S-parameter measurement results, made on three test samples SAV-581+, using HP8510B with a TRL calibration, from 0.1 to 18 GHz. Figure 5 shows the noise parameters measured on SAV-581+ devices from 0.5 to 6 GHz. Clockwise, from upper left, are the minimum noise figure, $F_{\text{min}}$ (dB), the 50 Ω noise figure $F_{50}$ (dB), $\Gamma_{\text{opt}}$ and equivalent noise resistance $R_n$. ■

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References